RATIONAL FUNCTIONS AND ASYMPTOTES NOTES

A **rational function** is a function that can be written as $f(x) = \frac{p(x)}{q(x)}$, where *p(x)* and *q(x)*are polynomials and $q(x) \neq 0$.

An example of a rational function would be $f(x) = \frac{3x^3+7x^2+2}{-4x^3+5}$.

Many rational functions contain **vertical asymptotes** and **end behavior asymptotes.**

A **vertical asymptote** is a value of x for which the function is not defined. As the value of x approaches the vertical asymptote, the value of the function rapidly approaches ∞ or -∞. Vertical asymptotes can be found by determining the values of x that make the denominator of the function equal zero and thus cause the function to be undefined.

An **end behavior asymptote** is a value or expression of y that the function approaches as x approaches ∞ or -∞. To find the end behavior asymptote of a function, we will refer to the following procedure:

Given $f(x) = \frac{p(x)}{q(x)} = \frac{a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_o}{bx^m + bx^{m-1} + b_{m-2}x^{m-2} + \cdots + b_1 x + b_o}$, where n is the degree of the numerator, m is the degree of the denominator, a_n is the leading coefficient of the numerator, and b_m is the leading coefficient of the denominator:

a) If $n < m$, then the x – axis (y = 0) is the end behavior asymptote.

b) If $n = m$, then the line $y = \frac{a_n}{b_m}$ is the end behavior asymptote.

c) If $n > m$, then the end behavior asymptote is an expression that is the quotient (not including the remainder) of p(x) and q(x).

In this section, we will analyze rational functions, and, thus be able to say:

1) I can determine the vertical asymptote(s), if there are any, of a rational function.

2) I can state the domain of a rational function.

3) I can use limits to describe the behavior of a rational function as x approaches a vertical asymptote.

4) I can determine the end behavior asymptote of a rational function.

5) I can use limits to describe the behavior of a rational function as x approaches -∞ or ∞.
FOR THE FOLLOWING FUNCTIONS, DETERMINE:

a) THE VERTICALASYMPTOTE(S), IF ANY, OF THE FUNCTION.

b) THE DOMAIN OF THE FUNCTION.

c) THE LIMITS OF THE FUNCTION AS x APPROACHES THE VERTICAL ASYMPTOTE(S).

d) THE END BEHAVIOR ASYMPTOTE OF THE FUNCTION.

e) THE LIMITS OF THE FUNCTION AS x APPROACHES -∞ AND ∞.

Example 1: $f(x) = \frac{3x^3+7x^2+2}{-4x^3+5}$

a) VERTICAL ASYMPTOTE(S):_______________ b) DOMAIN:_________________________________

c) $\lim_{x \to VA^-} f(x) =$ $\lim_{x \to VA^+} f(x) =$

d) END BEHAVIOR ASYMPTOTE:______________

e) $\lim_{x \to -\infty} f(x) =$ $\lim_{x \to \infty} f(x) =$

Example 2: $f(x) = \frac{2x}{3x^2+1}$

a) VERTICAL ASYMPTOTE(S):_______________ b) DOMAIN:_________________________________

c) $\lim_{x \to VA^-} f(x) =$ $\lim_{x \to VA^+} f(x) =$

d) END BEHAVIOR ASYMPTOTE:______________

e) $\lim_{x \to -\infty} f(x) =$ $\lim_{x \to \infty} f(x) =$

Example 3: $f(x) = \frac{4x^3}{x^2+2x-15}$

a) VERTICAL ASYMPTOTE(S):_______________ b) DOMAIN:_________________________________

c) $\lim_{x \to -5^-} f(x) =$ $\lim_{x \to -5^+} f(x) =$

$\lim_{x \to 3^-} f(x) =$ $\lim_{x \to 3^-} f(x) =$

d) END BEHAVIOR ASYMPTOTE:______________

e) $\lim_{x\to-\infty} f(x) =$ $\lim_{x\to\infty} f(x) =$

Example 4: $f(x) = \frac{x^3-3x^2+3x+1}{x-1}$

a) VERTICAL ASYMPTOTE(S):_______________ b) DOMAIN:__________________________________

c) $\lim_{x\to VA^-} f(x) =$ $\lim_{x\to VA^+} f(x) =$

d) END BEHAVIOR ASYMPTOTE:______________

e) $\lim_{x\to-\infty} f(x) =$ $\lim_{x\to\infty} f(x) =$